VECTOR - SPACES:

Defn: Let V be a non-empty set, whose elements are called VECTORS. Let F be the field whose elements are called SCALARS. Then $(V, \oplus, \odot)$ is said to be a Vector Space if it satisfies the following conditions.

1. $(V, \oplus)$ is an abelian group. (internal operation)

(i) $u \oplus v \in V \quad \forall\ u, v \in V$

(ii) $(u \oplus v) \oplus w = u \oplus (v \oplus w)$ (Associative law) $\forall\ u, v, w \in V$

(iii) $\exists\ e \in V$ s.t. $u \oplus e = e \oplus u = u \quad \forall\ u \in V$ (identity element)

(iv) $(u \oplus u^{-1}) = u^{-1} \oplus u = e$
$\forall\ u\ \exists\ u^{-1} \in V$ (existence of inverse)

(v) $(u \oplus v) = (v \oplus u)$ (commutative law).

2. It is closed under scalar multiplication. (External operation)
$a \cdot u \in V \quad \forall\ a \in F,\ u \in V.$

(i) $a(u+v) = au + av \quad \forall\ a \in F$ and $u, v \in V$ (Distributive law).

(ii) $(a+b)u = au + bv \quad \forall\ a, b \in F$ and $u \in V$

(iii) $a(bu) = (ab)u \quad \forall\ a, b \in F$ and $u \in V$

(iv) $1 \cdot u = u \quad \forall\ 1 \in F,\ u \in V.$
(existence of unit element)